Digital Signal Processing

WS 2017 Lab Sheet 3

Due date: 18.11.2017

Exercise 1: System properties — 5 Points

Determine if the following time discrete systems are memoryless (M), stable (S), causal (C), linear (L), and time invariant (TI). Give reasons for your answers.

a. $T(x[n]) = \cos(\pi n)x[n]$, (1)

b. $T(x[n]) = x[-n]$, (1)

c. $T(x[n]) = x[n-2] - 2x[n-17]$, (1)

d. $T(x[n]) = \Re(x[n])$, (1)

e. $T(x[n]) = \sum_{k=-n}^{n} (x[k])^2$ (1)

Exercise 2: Geom. series and other convenient formulas — 6 Points

Prove the following equations:

a. $\sum_{n=0}^{N-1} x^n = \begin{cases} N & x = 1 \\ \frac{1-x^N}{1-x} & x \in \mathbb{C}^{\neq 1} \end{cases}$

b. $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$, for $\{x \in \mathbb{C} \,|\, |x| < 1\}$

c. $\sum_{n=0}^{\infty} nx^n = \frac{x}{(1-x)^2}$, for $\{x \in \mathbb{C} \,|\, |x| < 1\}$
(Hint: Compute $\frac{\partial}{\partial x}$ on geometric series in b).

Exercise 3: Convolution — 7 Points

a. A linear and time invariant system has an impulse response $h\,[n]$. It is known that $h\,[n]$ vanishes outside the interval $N_0 \leq n \leq N_1$. An input sequence $x\,[n]$ vanishes outside the interval $N_2 \leq n \leq N_3$. Find the interval $N_4 \leq n \leq N_5$ which the output signal is confined to. Use the demo http://www.jhu.edu/signals/discreteconv2/index.html to visually verify your result (no submission necessary for this demo). (1)

b. Compute the convolution of $x[n] = (0.4)^n$ and $h\,[n] = nu\,[n]$. (3)

c. Given the following two sequences

$$x[n] = [3, 11, 7, 0, -1, 4, 2]\,, \qquad -3 \leq n \leq 3$$
$$h[n] = [2, 3, 0, -5, 2, 1]\,, \qquad -1 \leq n \leq 4\,,$$

determine the convolution $y[n] = x[n] * h[n]$ both by hand (for at least three n values), and using the matlab function `conv`. Create a figure with three subplots and plot x, h, and y using the function `stem`. (3)

Maximal score: 18 Points