Digital Signal Processing

WS 2017 Lab Sheet 6

Due date: 09.12.2017

Exercise 1: LTI discrete-time system — 12 Points

An LTI discrete-time system has frequencey response given by

$$H(e^{jw}) = \frac{(1 - je^{-jw})(1 + je^{-jw})}{1 - 0.8e^{-jw}} = \frac{1 + e^{-j2w}}{1 - 0.8e^{-jw}} = \frac{1}{1 - 0.8e^{-jw}} + \frac{e^{-j2w}}{1 - 0.8e^{-jw}}$$

a. Use one of the above forms of the frequency response to obtain an equation for the impulse response $h[n]$ of the system. (4)

b. From the frequency response, determine the difference equation that is satisfied by the input $x[n]$ and the output $y[n]$ of the system. (4)

c. If the input to this sytem is

$$x[n] = 4 + 2cos(\omega_0 n) \quad for - \infty < n < \infty$$

for what value of ω_0 will the output be of the form $y[n] = A = constant$ for $-\infty < n < \infty$? What is the constant A? (4)

Exercise 2: Discrete Time Fourier Transform II — 8 Points

a. Determine the Fourier transform of the sequence

$$r[n] = \begin{cases} 1, & 0 \leq n \leq M, \\ 0, & \text{otherwise} \end{cases}$$

(2)

b. Consider the sequence

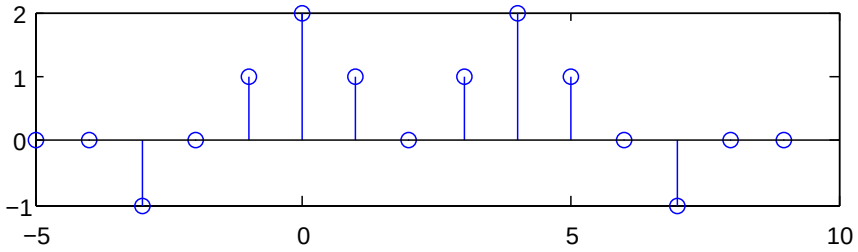
$$w[n] = \begin{cases} \frac{1}{2}\left(1 - \cos\left(\frac{2\pi n}{M}\right)\right), & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$$

Sketch $w[n]$ and express $W(e^{j\omega})$, the Fourier transform of $w[n]$, in terms of $R(e^{j\omega})$, which is the Fourier transform of $r[n]$. (4)

c. Plot the magnitude of $R(e^{j\omega})$ and $W(e^{j\omega})$ for $M = 4$. (2)

Exercise 3: Properties of Fourier transform — 8 Points

Let $X\left(e^{j\omega}\right)$ be the Fourier transform of the signal $x[n]$, which is plotted below. Answer the following questions without computing $X\left(e^{j\omega}\right)$ directly:



a. Compute $X\left(e^{j\omega}\right)|_{\omega=0}$. (1)

b. Compute $X\left(e^{j\omega}\right)|_{\omega=\pi}$. (1)

c. Compute $\int_{-\pi}^{\pi} X\left(e^{j\omega}\right) d\omega$. (2)

d. Determine and sketch the signal, whose Fourier transform is $X\left(e^{-j\omega}\right)$ (2)

e. Determine and sketch the signal, whose Fourier transform is $\mathrm{Re}\left(X\left(e^{j\omega}\right)\right)$ (2)

Maximal score: 28 Points